PERIODICO di MINERALOGIA Volume 88, No. 2, 2019

A Dynamic Kinetic Model of the Artificial Activated Sludge Process

S. FLESCHIN*, M. SCRIPCARIU**, A. A.BUNACIU*

**Department of Analytical Chemistry, Faculty of Chemistry, University of Bucharest, Sos.Panduri, Bucharest 5, Romania.*
***Drug Development Laboratory, Sos.Dudului, Nr.37, Bucharest,6, Romania.*

Abstracts

A kinetic model has been developed which describes the dynamic response of activated sludge to changes in substrate concentration. The well known phenomenon of "growth – rate hysteresis" can be explained by the simple yet biologically reasonable hypotheses of the model. Experimental results have verified the model quantitatively.

Introduction

Monod's (1) formulation of the growth of bacterial cultures has been extensively applied to the activated sludge process. Under steady conditions, the Monod model has proven to be a reasonably accurate description of the process (2, 3).

The Monod model has the following features:

1)the growth rate of a bacterial culture is given by

$$\frac{dX}{dt} = \mu X \qquad (1)$$

where X is the concentration of organisms and μ is the specific growth rate.

2)In the case of nutrient limited cultures, the specific growth rate is given by the expression

$$\mu = \mu_{max} [S / K_s + S)] \qquad (2)$$

where μ_{max} is the maximum specific growth rate, K_S is a saturation constant , and S is the substrate concentration.

3)An additional equation relates growth rate to substrate utilization

$$\frac{dX}{dt} = - Y \frac{dS}{dt} \qquad (3)$$

where Y is the yield coefficient. Thus, the rate of substrate utilization is given by

$$\frac{dS}{dt} = - \frac{1}{Y} \mu_{max} \frac{XS}{K_s} + S \qquad (4)$$

The Monod model was developed as a description of a steady state system. Under dynamic conditions, many investigators have recognized that the specific growth rate lags behind the values predicted by the Monod model in a response to changes in the substrate concentration. Perret (4) has referred to this phenomenon as "growth – rate hysteresis".

Several dynamic models which attempt to explain the growth – rate lag of biological cultures have been proposed. For instance, Young et al. (5) created a model based on the assumptions that when the substrate concentration is increased, the active transport of nutrients across the cell membrane introduces a time delay, and that there is a pure time delay between the increase in substrate concentration and the increase in enzymes which are required to react the substrate at a higher rate. Storer and Gaudy (6) constructed a dynamic model in which they assumed that the growth rate and the yield factor vary with time. Experiments showed that this view was consistent with observations. Other dynamic models have been proposed by Ramkrishna et al. (7) and Tanner (8). Experimental studies of the dynamic response of activated sludge have been conducted by Adams and Eckenfelder (9) and Sherrard and Lawrence (10).

The kinetic model presented in this paper is based on the proposition that the rate of metabolism is primarily controlled by the enzyme concentration in the biological mass. The enzyme concentration depends on both substrate concentration and time.

Furthermore, for any given substrate concentration there exists an optimum level of enzymes required to metabolize the substrate efficiently. In response to changes in substrate concentration, the enzyme concentration changes in a prescribed way to reach the optimum level at equilibrium. This qualitative view agrees generally with descriptions of the dynamic response expressed by Monod and later by others. In the following, these ideas are expressed quantitavely and the results are compared to experimental observations. Excellent agreement between theory and experiment has been found.

Theoretical Model

For the theoretical model are four hypotheses proposed:

1)The rate of substrate utilization is controlled by the concentration of enzymes in the biological mass ,

$$\frac{dS_u}{dt} = - A\ X\ z\ (S, t) \qquad (5)$$

where *A* is a rate constant, *z* is the specific concentration of enzymes in the cells, and the subscript *u* is used to denote utilization by the biomass, in contrast to the total change in substrate concentration which may include feed from external sources.

2)Enzymes decay during metabolism at a rate which is proportional to the specific concentration of enzymes in the system

$$\left(\frac{dz}{dt}\right)_{decay} = - k_1 z \qquad (6)$$

where k_1 is a rate constant, therefore, from eqs. (5) and (6)

$$\left(\frac{dz}{dt}\right)_{decay} = \frac{k_1}{AX}\ \frac{dS_u}{dt} \qquad (7)$$

3)For any given concentration of substrate, there exists an optimum concentration of enzymes, z_{equil}, which is reached at equilibrium. Metabolic regulatory mechanisms(11) control enzyme synthesis such that if the enzyme concentration is less than or equal to the equilibrium level, then enzyme synthesis is given by

$$\left(\frac{dz}{dt}\right)_{syn} = k_2\ z_{equil} \qquad z \le z_{equil} \qquad (8)$$

where k_2 is a rate constant.

Furthermore, if the enzyme concentration is greater than the equilibrium level, then enzyme synthesis ceases,

$$\left(\frac{dz}{dt}\right)_{syn} = 0 \qquad z > z_{equil} \tag{9}$$

At equilibrium the enzyme synthesis equals decay, thus $k_1 = k_2 = k$, where k is the rate constant. Therefore, the total rate of change of enzyme concentration is given by

$$\frac{dz}{dt} = k\,(z_{equil} - z) \qquad z \le z_{equil}$$

$$\frac{dz}{dt} = -k\,z \qquad z > z_{equil} \tag{10}$$

4)The equilibrium level of enzyme concentration is given by

$$z_{equil} = z_{max}\, S / (K_s + S) \tag{11}$$

where z_{max} is the maximum possible specific enzyme concentration, K_S is a saturation constant, and S is the total substrate concentration.

Note that under steady conditions $z = z_{equil}$, therefore, from eq. (5), the rate of substrate utilization is given by

$$\left(\frac{dS_u}{dt}\right)_{equil} = -A\,\frac{z_{max} S}{K_s + S}\,X \tag{12}$$

which is the familiar Monod eq. (4) where $Az_{max} = \mu_m/Y$.

Thus, this theoretical model gives results which agree with accepted models of the steady state.

If there is no external feed to the system, then the substrate utilization is equal to the total rate of change of substrate concentration

$$\frac{dS_u}{dt} = \frac{dS}{dt} \tag{13}$$

If there is feed to the system

$$\frac{dS_u}{dt} = \frac{dS}{dt} - \frac{df}{dt} \tag{14}$$

where f is the concentration of substrate fed to the system.

Preliminary Results and Example for Glucose

From the theoretical model, a differential equation describing the substrate concentration can be derived. The derivative of eq. (5) combined with eq (10) gives

$$\frac{d^2 S_u}{dt^2} - \frac{1}{X}\frac{dS_u}{dt}\frac{dX}{dt} + k\,\frac{dS_u}{dt} + kX\,\frac{\mu_m}{Y}\frac{S}{K_S + S} = 0 \qquad z \le z_{equil} \tag{15}$$

and

$$\frac{d^2 S_u}{dt^2} - \frac{1}{X}\frac{dS_u}{dt}\frac{dX}{dt} + \frac{dS_u}{dt} = 0 \qquad z > z_{equil}$$

If there are no external sources or losses of the biological mass, dX/dt in the second term can be replaced by $-Y\,dS_u/dt$. A general analytical solution of this equation is difficult. However, it can easily be solved by numerical methods.

With simplifying approximations, analytical solutions to certain special cases can be found. Under normal conditions, the second term in eq. (15) is small and can be safely

ignored. If, in addition the substrate concentration is small ($S << K_S$) then eq. (15) becomes linear:

$$\frac{d^2 S_u}{dt} + k\frac{d}{d}\frac{S}{t} + kX\frac{\mu_m}{YK_s}S = 0 \qquad z < z_{equil} \qquad (16)$$

$$\frac{dS_u}{dt} + k\frac{dS_u}{dt} = 0 \; z > z_{equil}$$

Calculation of the yield factor Y in utilization of glucose in artificial sludge.

The yield factor Y can determined directly from the CO_2 production data. The reaction of converting glucose in CO_2 is :

$$C_6H_{12}O_6 + 6\,O_2 \Rightarrow 6\,CO_2 + 6\,H_2O$$

Thus 180 g glucose will produce 264g CO_2.

Then the CO_2 produced by the utilization of substrate should be

$$\left(\frac{dCO_2}{dt} - E\right) = -(1 - Y)\frac{264}{180}\frac{dS_u}{dt}$$

Earlier it was found that

$$\frac{dS_u}{dt} = -2{,}5\left(\frac{dCO_2}{dt}\right) + 2{,}5E \quad \text{which implies that}$$

$(1\text{-}Y) = 180 / (2{,}5 \times 264) = 0{,}27$. Or $Y = 0{,}73$.

The yield factor calculations are based on the mass of the substrate. It is more common to use the chemical oxygen demand (COD) equivalent of the substrate. The calculated yield factor is related to the COD yield factor by

$$Y_{COD} \cong 1{,}07\; Y = 0{,}78.$$

Selective References

1. J. MONOD, *Ann.Rev.Microbiol*, **3**, 371, (1949).
2. G. L. JONES, *Water.Res*, **7**, 1475, (1993).
3. E. A. PEARSON, *Kinetics of biological treatment in Advances in Water Quality Improvement*, E. Y. GLOYNA, Eds. Texas. U.P, pp.381-394, 1986.
4. C. J. PERET, *J.Gen.Microbiol* , **22**, 589, (1996).
5. T. B. YOUNG, D. F. BRUELY, H. R. BUNGAY, *Biotechnol.Bioeeng*, **12**, 747, (1997).
6. F. F. STORER, A. F. GANDY, *Jr.Environ.Sci.Technol*, **3,** 143, (1986).
7. D. RAMKRISHA, A. G. FREDRICKSON, H. M. TSUCHYA, *Biotechnol.Bioeeng*,9, 129, (1989).
8. R. D. TANNER, , *Biotechnol.Bioeeng*, **12**, 831, (1987).
9. C. E. ADAMS, W. W. ECKENFENDER, *J.San.Eng.Div.Proc*, **96**, 333, (1992).
10. J. M. SHERRARD, A. W. LAWRENCW, *J.Water.Pollut.Control.Fed*, **47**, 1848, (1990).
11. P. R. DUGAN, Biochemical Ecology of Water Pollution, A Plenum Rosetta Edition, New York , (1985).
12. Lydia Maria Vaicum, Epurarea apelor uzate cu namol activ, Bazele biochimice, Editura Academiei R.S.R, 1981.