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A PHENOMENOLOGICAL FRAMEWORK FOR REGIONAL SEISMICITY USING MANTLE-FLOW INFORMATION

Physical Definitions, Numerical Consistency and a Validation Protocol

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ABSTRACT: Mantle circulation contributes to plate motion and lithospheric deformation, motivating investigation of mantle-flow information as an additional covariate in regional seismicity models. This physical connection does not, however, establish that mantle vorticity measures fault stress or improves earthquake forecasts. This paper develops a phenomenological framework that distinguishes mantle material velocity from seismic-wave velocity, rotation from deformation, and descriptive scores from calibrated occurrence probabilities. A regional vorticity descriptor is defined by spatially averaging the magnitude of the curl of a specified mantle-flow reconstruction and normalising the result using training data. Tectonic loading and historical seismicity are represented by separately defined covariates. Numerical material from the preliminary formulation is retained and examined, including five historical earthquake examples, regional comparison tables, four exponential-mapping coefficients and an Assam–Arunachal worked example. Recalculation gives a composite product of 1.79634 and an exponential output of 98.8789 for the original example. However, its assigned Vortex Strength Index of 1.95 is incompatible with a 0–1 normalisation. A bounded composite index with mapping coefficient 2.50 cannot produce an output above 91.7915 on a 0–100 scale. These findings establish internal numerical constraints rather than forecasting skill. A separate event-count formulation and validation protocol are proposed to test whether mantle-flow information improves predictions beyond tectonic and historical-seismicity baselines. Earlier regional percentages and future magnitude intervals are retained only as unvalidated entries from the preliminary

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manuscript. No observational calibration, independent predictive validation or operational hazard assessment is claimed.

Keywords: mantle flow; vorticity; regional seismicity; tectonic loading; earthquake forecasting; model calibration; uncertainty; seismic hazard.

1. INTRODUCTION

Earthquake occurrence depends on fault loading, frictional resistance, crustal structure, fluid pressure and stress redistribution. Mantle circulation affects the broader tectonic system through plate motion and lithospheric deformation [1,2]. This connection provides a reason to investigate mantle-derived descriptors within regional seismicity models.

The scientific question is whether a mantle-flow descriptor adds information beyond plate kinematics, fault properties, geodetic deformation and historical seismicity. A correlation between mantle circulation and tectonically active regions does not, by itself, establish an improvement in forecasting.

Three quantities must remain distinct. A descriptive index combines selected variables into a numerical score. An earthquake occurrence forecast specifies an expected rate or probability for a defined region, magnitude range, depth range and time interval. Probabilistic seismic hazard assessment estimates ground-motion exceedance at a site using source information, ground-motion relationships and their uncertainties [3,4].

The present paper concerns descriptive indices and a proposed occurrence model. It does not calculate ground-motion exceedance or societal risk.

The preliminary formulation combined a Vortex Strength Index, a Tectonic Factor and a Historical Weight through an exponential mapping. It also presented historical-event examples and regional percentages. These numerical entries are retained for examination, but their preservation does not establish observational provenance or statistical calibration.

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The contribution is a corrected physical definition of the mantle descriptor, a numerical check of the preliminary calculations and a validation design capable of testing incremental usefulness. No new earthquake-generation law or successful forecast is demonstrated.

2. SCIENTIFIC BACKGROUND

2.1. Mantle circulation and plate motion

Mantle material deforms and flows over geological timescales. Its motion depends on buoyancy, viscosity, boundary conditions and interaction with the lithosphere. Plate motion and mantle circulation are dynamically coupled [1,2].

If observed plate velocities are imposed as boundary conditions in a mantle-flow reconstruction, the calculated flow already contains plate-kinematic information. A later association between that flow and earthquake occurrence may partly reproduce the imposed tectonic information.

The mantle descriptor, convergence rate and historical seismicity must therefore not be treated as independent predictors without an analysis that examines their shared information.

2.2. Tomography and mantle material velocity

Seismic tomography estimates variations in seismic-wave velocity. Mantle material velocity describes the much slower motion of the medium through which those waves travel.

A seismic-wave velocity anomaly cannot be inserted directly into a material-flow vorticity calculation. A mantle-flow reconstruction requires additional assumptions concerning density, temperature, composition, viscosity and mechanical boundary conditions.

S40RTS is an example of a mantle shear-wave velocity model [6]. SEMUCB-WM1, GyPSuM and MIT-P08 were also named in the preliminary manuscript as potential structural inputs. No numerical extraction or flow inversion using these models is reported here.

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2.3. Earthquake catalogues and forecast evaluation

Earthquake counts depend on region boundaries, observation duration, magnitude threshold, detection capability and the treatment of clustered events.

Magnitude completeness must be assessed before regional counts are compared [7]. Sequences containing aftershocks generally require treatment beyond an independent-event approximation [8].

Forecast evaluation requires specified target events and test conditions [5]. Selecting several well-known earthquakes after their occurrence cannot establish predictive skill. An evaluation must also include regions and intervals without qualifying events.

3. AVAILABLE NUMERICAL MATERIAL AND ITS STATUS

The available numerical material consists of values printed in the preliminary manuscript. It does not constitute a complete observational modelling dataset.

Table 1. Available material and its evidential status

Material	Information available	Evidential status
Historical earthquake examples	Five events, years, magnitudes, percentages and classifications	Historical illustrations; model labels unvalidated
Earlier regional assessment	Five regions, percentages, magnitude intervals and the 2027–2032 window	Earlier forecast entries retained for audit; not endorsed forecasts
India comparison	Five region labels, percentages and ranks	Unvalidated ordering
Global comparison	Five region labels, percentages and	Unvalidated ordering

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Material	Information available	Evidential status
	ranks	
Assam–Arunachal example	VSI = 1.95; TF = 0.98; HW = 0.94; $\lambda = 2.50$	Assigned inputs permitting an arithmetic check
Mapping coefficients	$\lambda = 0.85$, 1.50, 2.50 and 3.50	Unfitted sensitivity parameters
Regional mantle-flow grids	Not supplied	Regional vorticity cannot be independently reproduced
Catalogue queries and completeness assessment	Not supplied	Historical weights cannot be independently reproduced
Regional stress and fault-locking estimates	Not supplied	Stress and fault-loading factors cannot be independently reproduced
Training and test predictions	Not supplied	Empirical forecasting performance cannot be calculated
ARIMA–LSTM implementation	Architecture, training details and results not supplied	Proposed extension rather than completed analysis

The preliminary text quoted an India–Eurasia motion rate of approximately 40–50 mm yr^{-1} and approximately 5 cm yr^{-1} elsewhere. These are broad background descriptions rather than documented local fault-normal convergence rates. They are not used as numerical inputs in the revised calculations.

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Public earthquake catalogues, GNSS observations, plate-motion models and volcanic records remain candidate inputs. Naming these sources does not demonstrate that they were downloaded, processed or used for calibration.

4. PHYSICAL AND MATHEMATICAL FORMULATION

Table 2. Symbols, definitions and interpretation

Symbol	Definition or role	Units or condition
$u(x,t)$	Mantle material-velocity vector	Length per time
ω	$\nabla \times u$	Inverse time
Ω	Magnitude of ω	Non-negative; inverse time
n	Specified reference normal	Unit vector
ω_n	$\omega \cdot n$	Signed component; inverse time
D	Symmetric strain-rate tensor	Inverse time
W	Antisymmetric rotation tensor	Inverse time
η	Dynamic viscosity	Pa s
τ	Deviatoric stress	Pa

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Symbol	Definition or role	Units or condition
V_i	Mantle averaging volume associated with region i	Volume
$q_i(x)$	Non-negative averaging weight	Positive volume integral required
Ω_i	Weighted regional vorticity magnitude	Inverse time
D_m	$d_{base} - d_{top}$	Positive depth interval
N_X	Training-based normalisation operator	Dimensionless output
VSI_i	Normalised regional vorticity descriptor	Between 0 and 1
C_i	Defined convergence-rate measure	Length per time
TF_i	Normalised tectonic factor	Between 0 and 1
R_i	Historical event count per area and time	Events per area per time
HW_i	Normalised historical-seismicity descriptor	Between 0 and 1
H_i	Weighted descriptive composite index	Between 0 and 1
λ	Exponential-mapping coefficient	Positive and dimensionless for dimensionless H
Q	Descriptive exponential output	Numerical scale from 0 towards 100

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Symbol	Definition or role	Units or condition
β_0, β, β_V	Log-rate model coefficients	Determined by model specification
μ_ij	Expected count in region i and interval j	Dimensionless expected count
ΔL	Difference between held-out log scores	Predictive comparison
Σ_z	Covariance matrix of index components	Defined for the specified components

4.1. Mantle material velocity

Let u(x,t) denote mantle material velocity in a specified reference frame:

u(x,t) = [u_x(x,t), u_y(x,t), u_z(x,t)]. (1)

Coordinates, units and reference frame must be stated. Spatial derivatives must use operators appropriate to the coordinate system or numerical mesh.

4.2. Vorticity

Define the vorticity vector:

ω = ∇ × u. (2)

Its magnitude is:

Ω = (ω · ω)^(1/2). (3)

A directional component may be defined as:

ω_n = ω · n. (4)

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If velocity is measured in metres per second and distance in metres, ω has units s^{-1}.

Neither sign of ω_n is assumed to increase or decrease seismicity. Counter-rotation is not interpreted as stress relaxation.

Clockwise and counter-clockwise descriptions require a stated viewing direction and reference normal. Comparisons between flow reconstructions also require consistent reference-frame conventions.

4.3. Rotation, deformation and stress

The velocity gradient can be decomposed as:

$$\nabla u = D + W. \quad (5)$$

The symmetric and antisymmetric components are:

$$D = [\nabla u + (\nabla u)^T]/2,$$

$$W = [\nabla u - (\nabla u)^T]/2. \quad (6)$$

For an incompressible Newtonian approximation:

$$\tau = 2\eta D. \quad (7)$$

Consider rigid rotation with constant angular velocity a:

$$u = a \times x,$$

$$\omega = 2a,$$

$$D = 0. \quad (8)$$

Equation (8) demonstrates that non-zero vorticity can coexist with zero deviatoric stress under Eq. (7) [1].

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Vorticity alone therefore cannot determine fault loading or earthquake probability. It is a candidate flow descriptor whose usefulness must be tested. Strain-rate and basal-traction descriptors should also be examined where sufficiently resolved model outputs are available.

4.4. Regional averaging

For mantle volume V_i associated with region i, define:

$$\Omega_i = [\int (V_i)\, q_i(x)\, \Omega(x)\, dV] / [\int (V_i)\, q_i(x)\, dV]. \quad (9)$$

The spatial weight q_i(x) must be non-negative, and its volume integral must be positive.

The averaging volume, depth interval, resolution and smoothing procedure must be specified before testing. A volume beneath a surface region is a sampling convention rather than proof of a purely vertical causal connection.

For depth measured positively downwards:

$$D_m = d_{base} - d_{top} > 0. \quad (10)$$

4.5. Normalisation and covariates

For training-sample limits a_X and b_X, with b_X > a_X, define:

$$N_X(x) = \min\{1, \max\{0, (x - a_X)/(b_X - a_X)\}\}. \quad (11)$$

The normalisation limits must be estimated using training data and then fixed for evaluation. Values outside the training range should be flagged even when clipping is applied.

Constant features and missing data require separately declared treatment. Missing measurements must not be interpreted as zero physical activity.

 https://doi.org/10.5281/zenodo.23215490

Define:

$$VSI_i = N_{\Omega}(\Omega_i),$$

$$0 \leq VSI_i \leq 1. \quad (12)$$

For a documented convergence-rate measure C_i in an appropriate convergent setting:

$$TF_i = N_C(C_i). \quad (13)$$

For a preceding historical interval:

$$R_i = N_i(M \geq M_{ref})/(A_i T_i),$$

$$HW_i = N_R(R_i). \quad (14)$$

Here A_i is region area, T_i is observation duration, and M_{ref} is a magnitude threshold compatible with the completeness assessment. Magnitude convention and depth limits must also be specified.

The historical interval must end before the forecast interval begins.

4.6. Descriptive composite index

Define:

$$H_i = w_V VSI_i + w_T TF_i + w_H HW_i,$$

$$w_V + w_T + w_H = 1,$$

$$w_V, w_T, w_H \geq 0. \quad (15)$$

It follows that:

$$0 \leq H_i \leq 1. \quad (16)$$

 https://doi.org/10.5281/zenodo.23215490

H_i is a descriptive score rather than an earthquake probability.

The weights are analysis choices unless estimated against a defined outcome. The additive form avoids automatically setting the entire score to zero when one component vanishes, but does not establish empirical superiority over models containing interactions.

Convergence alone is not an appropriate universal tectonic predictor for transform, extensional and intraplate settings.

5. NUMERICAL CONSISTENCY OF THE PRELIMINARY FORMULATION

5.1. Preliminary expressions

The earlier expressions were:

$$VSI_{old} = \omega_c C_r S_f / D_m,$$

$$TF_{old} = C_r F_s,$$

$$H_{old} = VSI_{old} \times TF_{old} \times HW. \quad (17)$$

If S_f, F_s and HW are dimensionless:

$$[\omega_c] = T^{-1},$$

$$[C_r] = LT^{-1},$$

$$[D_m] = L.$$

Consequently:

$$[VSI_{old}] = T^{-2},$$

$$[TF_{old}] = LT^{-1},$$

 https://doi.org/10.5281/zenodo.23215490

$[H_old] = LT^{-3}$.

The original construction is therefore not dimensionless under these definitions.

Furthermore, H_old is proportional to C_r^2, so convergence enters twice. Such repeated weighting requires physical or empirical justification.

If H_old retains these dimensions, its exponential coefficient must have compatible dimensions. A bare numerical coefficient cannot be transferred between unit conventions without accounting for that dependence.

5.2. Exponential mapping

For dimensionless $H \geq 0$ and dimensionless $\lambda > 0$:

$$Q(H; \lambda) = 100[1 - \exp(-\lambda H)]. \quad (18)$$

The first derivative is:

$$dQ/dH = 100\lambda \exp(-\lambda H) > 0. \quad (19)$$

The second derivative is:

$$d^2Q/dH^2 = -100\lambda^2 \exp(-\lambda H) < 0. \quad (20)$$

For a bounded index:

$$0 \leq Q \leq 100[1 - \exp(-\lambda)], \text{ when } 0 \leq H \leq 1. \quad (21)$$

These relationships establish monotonicity, concavity and numerical bounds. They do not establish empirical calibration or physical stability.

Table 3. Earlier coefficient choices and ceilings for a bounded index

 https://doi.org/10.5281/zenodo.23215490

	Earlier descriptive label	Maximum Q for $H \leq 1$
.85	Low hazard	57.2585
.50	Moderate hazard	77.6870
.50	High hazard	91.7915
.50	Extreme hazard	96.9803

The earlier labels are retained as records of the preliminary specification. No fitted evidence establishes these hazard categories.

Assigning a higher λ solely because a region has already been labelled high hazard, and then using the resulting output to confirm that label, introduces circular reasoning.

A common λ preserves the ranking of H exactly. Region-specific coefficients can alter rankings and require separate justification.

5.3. Assam–Arunachal worked example

Table 4. Assigned inputs and arithmetic checks

Quantity	Assigned or calculated value

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Quantity	Assigned or calculated value
VSI_old	1.95
TF_old	0.98
HW	0.94
λ	2.50
$H_{old} = 1.95 \times 0.98 \times 0.94$	1.79634
$Q_{old} = 100[1 - \exp(-2.50 \times 1.79634)]$	98.8789
Maximum Q for $H \leq 1$ and $\lambda = 2.50$	91.7915
H required for Q = 98.9 at $\lambda = 2.50$	1.8039
H required for Q = 99.0 at $\lambda = 2.50$	1.8421

The reported output of approximately 98.9 is arithmetically consistent with the assigned inputs. It is not an established earthquake probability.

VSI_old = 1.95 lies outside the proposed 0–1 normalisation. A bounded H with $\lambda = 2.50$ cannot yield 98.9.

The inverse relationship is:

$$H_{required} = -\ln(1 - Q/100)/\lambda. \quad (22)$$

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The difference between the product 1.79634 and the inverse value calculated from 98.9 reflects rounding of the output.

The original assigned inputs cannot be treated as reconstructed physical observations without their sources and processing methods. Clipping VSI_old to one would not recover that missing information.

6. PAST EVENTS AND EARLIER FUTURE ENTRIES

6.1. Historical earthquake examples

Table 5 lists events that have already occurred. The percentages are unvalidated preliminary model labels rather than demonstrated hindcast performance.

Table 5. Historical earthquakes and unvalidated preliminary outputs

Event	Year	Region	Magnitude printed in preliminary manuscript	Earlier percentage label	Earlier classification
Sumatra–Andaman	2004	Indonesia and adjoining region	9.1	98.9%	Very high
Kashmir	2005	Kashmir	7.6	97.1%	Very high
Maule	2010	Chile	8.8	98.7%	Very high
Tohoku	2011	Japan	9.0	99.0%	Very high
Gorkha	2015	Nepal	7.8	97.6%	Very high

The USGS records consulted list M9.1 for Sumatra–Andaman [11], M7.6 for the 8 October 2005 Pakistan earthquake [13], M9.1 for Tohoku [12] and M7.8 for Gorkha [14].

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The preliminary manuscript printed M9.0 for Tohoku. That value is retained in Table 5 to document the original numerical material. An empirical analysis must use and report a consistent catalogue version.

These selected cases cannot establish accuracy or a false-alarm rate. A defined evaluation population, including non-event intervals, is required [5].

6.2. Earlier future entries

Table 6 preserves entries proposed for the future window 2027–2032. Their forecast interpretation is withdrawn in the revised framework.

Table 6. Unvalidated earlier future entries retained for audit

Region	Earlier percentage label	Earlier proposed magnitude interval	Earlier proposed window	Present interpretation
Assam–Arunachal	98.9%	M7.9–8.4	2027–2032	Unvalidated record; not an operational warning
Uttarakhand–Himachal	98.0%	M7.6–8.1	2027–2032	Unvalidated record; not an operational warning
Andaman–Nicobar	97.3%	M7.2–7.8	2027–2032	Unvalidated record; not an operational warning
Japan Trench	99.0%	M8.0–8.7	2027–2032	Unvalidated record; not an operational warning

 https://doi.org/10.5281/zenodo.23215490

Region	Earlier percentage label	Earlier proposed magnitude interval	Earlier proposed window	Present interpretation
Sumatra–Indonesia	98.9%	M8.0–8.8	2027–2032	Unvalidated record; not an operational warning

No calculation supplied with the preliminary manuscript establishes these magnitude intervals or selects the 2027–2032 window.

These entries must not be interpreted as evidence that the listed earthquakes will occur. Retaining them for transparency does not make them valid forecasts.

The corrected formulation does not generate the entries in Table 6.

6.3. Earlier India ordering

Table 7. Earlier India ordering without validated hazard interpretation

Region	Earlier percentage label	Earlier rank
Assam–Arunachal	98.9%	1
Uttarakhand–Himachal	98.0%	2
Andaman–Nicobar	97.3%	3
Nepal–India Border	96.8%	4
Himachal, Kangra region	95.9%	5

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These entries do not establish a validated regional hazard ranking.

The Uttarakhand–Himachal and Kangra definitions may overlap. Treating overlapping regions as independent observations would distort comparisons and uncertainty estimates.

An empirical study must specify region polygons, source zones and overlap rules before calculating rankings.

6.4. Earlier global ordering

Table 8. Earlier global ordering without validated hazard interpretation

Region	Country or regional designation	Earlier percentage label	Earlier rank
Japan Trench	Japan	99.0%	1
Sumatra–Indonesia	Indonesia	98.9%	2
Chile Subduction Zone	Chile	98.7%	3
Alaska–Aleutian Trench	United States region	98.2%	4
Peru–Chile Trench	Peru–Chile region	97.8%	5

The Chile Subduction Zone and Peru–Chile Trench labels may describe overlapping parts of a tectonic system. Their spatial definitions must be made explicit.

No uncertainty is supplied for these values. Differences such as 99.0 versus 98.9 cannot establish meaningful differences in hazard.

6.5. Interpretation of retained numerical material

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The retained tables document the numerical claims in the preliminary manuscript and permit examination of their internal consistency.

They do not establish a measured mantle-vorticity field, a fitted relationship with earthquake occurrence, calibrated regional probabilities, validated rankings or future magnitude intervals.

Parameters in a revised empirical model should not be selected solely to reproduce the earlier percentages.

7. PROPOSED EMPIRICAL MODEL AND VALIDATION

7.1. Target-event definition

Let Y_ij count qualifying earthquakes in region i during interval j. The magnitude threshold, depth range, region boundaries, catalogue and duration must be specified before fitting.

For region area A_i, interval duration Δt_j and event rate r_ij per unit area per unit time:

$\mu_{ij} = A_i \Delta t_j r_{ij}$. (23)

The quantity μ_{ij} is the expected count for the specified exposure.

Area-based exposure is a modelling choice. Fault-based source models may require a different representation.

7.2. Baseline and mantle-augmented models

Let x_ij represent baseline tectonic, fault and historical-seismicity covariates. Using a fixed reference rate r_ref with the same units as r_ij:

M0: $\ln(r_{ij}/r_{ref}) = \beta_0 + \beta^T x_{ij}$. (24)

M1: $\ln(r_{ij}/r_{ref}) = \beta_0 + \beta^T x_{ij} + \beta_V VSI_i$. (25)

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The models are fitted separately using the same training observations. The coefficient β_V is not constrained to be positive.

The baseline should represent credible existing explanations. It should not be weakened to make the additional mantle term appear useful.

Both models must use comparable fitting procedures and identical evaluation targets.

If a conditional Poisson model is appropriate:

$$Y_{ij} \sim \text{Poisson}(\mu_{ij}),$$

$$P(Y_{ij} \geq 1) = 1 - \exp(-\mu_{ij}). \quad (26)$$

Equation (26) defines a model probability for a specified event population and exposure. It does not demonstrate empirical calibration.

Clustering, overdispersion and non-stationarity may require a different count or point-process model [8]. The probability of zero events must then be calculated from that model.

7.3. Required data documentation

An empirical implementation must document the catalogue source, extraction date, query parameters, event identifiers, duplicate handling and magnitude conversions.

It must report magnitude completeness, depth filters, target-event definitions, region polygons, areas and overlapping boundaries.

Historical feature intervals must be distinguished from outcome intervals.

The mantle reconstruction must specify its physical assumptions, reference frame, viscosity structure, boundary conditions, numerical resolution and derivative calculation.

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Plate-motion, geodetic and fault-data sources must be identified precisely. These are requirements for future empirical work rather than completed analyses reported here.

7.4. Information leakage and test design

Feature normalisation, smoothing, imputation and parameter selection must be fitted within training partitions.

Later earthquake outcomes must not influence the construction of earlier historical features.

A modern mantle reconstruction may support retrospective association analysis, but it cannot automatically be represented as information available to an operational forecast at an earlier date.

Temporal holdouts and spatial transfer tests answer different questions. Both are relevant when a largely static mantle descriptor is used.

The validation design must account for spatial and temporal dependence [9]. If test outcomes are used to revise the model, a new independent evaluation is needed.

7.5. Predictive scores

For held-out observations under the conditional Poisson model:

$$L = \Sigma_{(i,j)} [Y_{ij} \ln(\mu_{ij}) - \mu_{ij} - \ln(Y_{ij}!)]. \quad (27)$$

Compare models using:

$$\Delta L = L(M1) - L(M0). \quad (28)$$

A positive realised ΔL indicates a better log score on that test set. Sampling uncertainty and consistency across tests must also be examined.

 https://doi.org/10.5281/zenodo.23215490

For binary occurrence, let $Z_{ij} = 1$ when $Y_{ij} \geq 1$ and zero otherwise. With predicted occurrence probability p_{ij}:

$$BS = (1/n) \Sigma_{(i,j)} (p_{ij} - Z_{ij})^2. \quad (29)$$

Here n is the number of evaluated region–interval observations. Lower Brier scores indicate better performance under this measure.

Calibration diagnostics should compare predicted probabilities with observed frequencies, including uncertainty. Discrimination measures alone do not establish calibration.

7.6. Uncertainty and robustness

For $H = w^T z$, with fixed weights and covariance matrix Σ_z:

$$Var(H) = w^T \Sigma_z w. \quad (30)$$

This expression includes covariance between the components. It does not automatically include uncertainty in estimated weights or normalisation limits.

Shared inputs can generate correlated errors. A complete analysis should propagate uncertainties in mantle reconstruction, viscosity, catalogue completeness, covariate construction and fitted parameters.

Robustness checks should examine alternative mantle models, reference frames, depth ranges, averaging volumes, resolutions, magnitude thresholds and baseline specifications.

An ensemble spread is not automatically a calibrated confidence interval.

7.7. Negative and inconclusive outcomes

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The usefulness of VSI would be unsupported if it produces negligible or negative held-out improvement, worsens calibration, or loses its benefit when a stronger tectonic or spatial baseline is introduced.

An unstable effect across plausible mantle reconstructions would weaken interpretation.

Uncertainty encompassing both useful and harmful performance indicates an inconclusive result. Such outcomes must be reported rather than removed through selective region or coefficient choices.

8. REGIONAL SCOPE AND FUTURE EXTENSIONS

8.1. Indian study regions

The preliminary manuscript identified the Western and Central Himalaya, Northeast India, Indo–Burma region, Shillong Plateau, Andaman–Nicobar, Kachchh and Delhi–Haridwar Ridge as areas of interest.

These labels describe possible study domains rather than a common statistical population. Collision, subduction and intraplate settings should not automatically share the same convergence-based predictor.

Shillong 1897, Kangra 1905, Bihar–Nepal 1934, Assam 1950 and Bhuj 2001 were included as historical context in the preliminary discussion. They are not additional validation results for the present framework.

8.2. Volcanic information

Volcanic activity is not a direct measurement of mantle vorticity. Any proposed volcanic predictor requires an explicit definition, source and physical rationale.

Its incremental contribution must be evaluated independently rather than assumed from geographical association.

8.3. Machine learning

No machine-learning result is reported.

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ARIMA–LSTM or other extensions would require documented architecture, training data, tuning procedures and independent comparisons against simpler baselines.

Added complexity is justified only by reproducible improvement.

8.4. Directional circulation

Counter-rotation is not retained as a demonstrated mechanism of stress relaxation.

A directional hypothesis requires a specified reference frame, local normal and physical model. The statistical test must permit either sign of association and account for additional model-selection freedom.

9. DISCUSSION

The numerical analysis distinguishes correct arithmetic from valid scientific inference.

The Assam–Arunachal example reproduces Q = 98.8789 from the printed inputs. However, VSI = 1.95 lies outside the proposed 0–1 range, and a bounded index with $\lambda = 2.50$ cannot exceed 91.7915.

A numerical range of 0–100 does not establish a calibrated occurrence probability.

Vorticity is not a direct measure of deformation or stress. Under $\tau = 2\eta D$, rigid rotation has $D = 0$ despite non-zero vorticity.

Mantle-flow reconstructions may already contain plate-kinematic information. Their association with seismicity may therefore duplicate information represented in a baseline model.

A slowly varying mantle descriptor may characterise persistent spatial differences while supplying little information about event timing.

The historical sample in Table 5 is selected and incomplete. The future entries in Table 6 record unsupported earlier forecast claims rather than results generated by the corrected model.

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The available material does not contain the physical feature dataset, training record or independent predictions required to establish forecasting skill.

The value of the revised framework lies in specifying a reproducible test and identifying the physical, numerical and statistical conditions that an empirical implementation must satisfy.

10. CONCLUSION

A revised framework has been specified for investigating mantle-flow information in regional seismicity modelling.

The mantle descriptor is defined from the curl of material velocity with explicit spatial support and normalisation. Tectonic and historical-seismicity components are represented separately, and their independence is not assumed.

All four historical and regional comparison tables from the preliminary formulation have been retained with their evidential status made explicit.

The corrected Assam–Arunachal product is 1.79634, producing an exponential output of approximately 98.8789 from the original assigned inputs. This verifies arithmetic only.

The input VSI = 1.95 is incompatible with 0–1 normalisation. For bounded H and $\lambda = 2.50$, Q cannot exceed 91.7915.

No validated future earthquake probability, magnitude interval or regional hazard ranking is established.

The next scientific step is an independent comparison of M0 and M1 using predictive scores, calibration diagnostics and uncertainty estimates. Positive, negative and inconclusive outcomes must all remain possible.

DATA AVAILABILITY

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The numerical material consists of the tables, assigned coefficients and worked-example inputs reproduced from the preliminary manuscript. No regional mantle-flow grid or empirical training-and-test dataset accompanies this analysis.

Selected historical magnitudes were checked against USGS records [10–14]. The arithmetic checks follow from Eqs. (18), (21) and (22), together with the stated worked-example inputs.

CODE AVAILABILITY

No trained forecasting model or empirical validation pipeline is reported. The numerical consistency checks use multiplication, exponentiation and logarithms and can be reproduced from the expressions and inputs printed in the manuscript.

CONFLICT OF INTEREST

The author declares no conflict of interest.

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